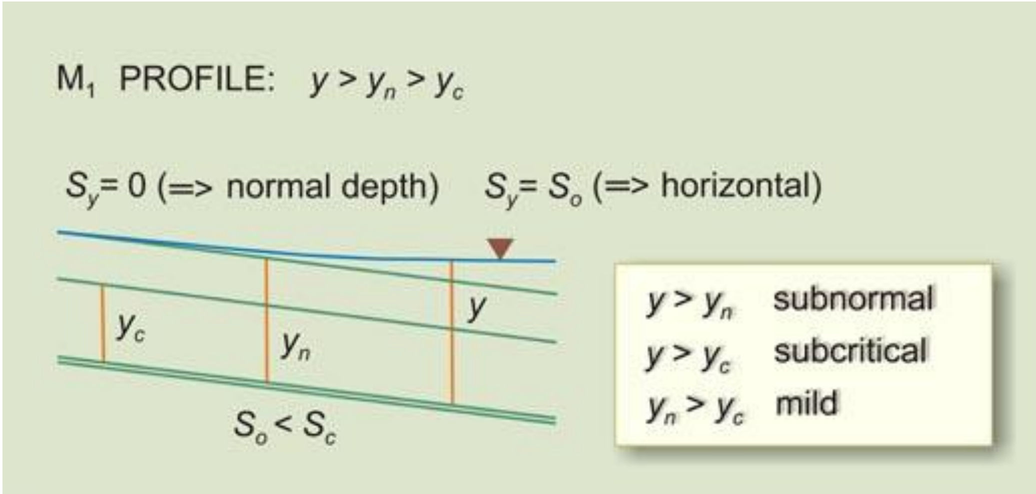


M_1 water-surface profile.

M_1 WATER-SURFACE PROFILE CALCULATED ONLINE

Victor M. Ponce

Professor Emeritus of Civil and Environmental Engineering

San Diego State University, San Diego, California

10 November 2023

ABSTRACT. An online calculation of an M_1 open-channel flow water-surface profile is shown in detail. Two examples using the script **ONLINE_WSPROFILES_21** demonstrate the utility of using this online digital tool for the accurate and expedient calculation of an M_1 water-surface profile.

1. INTRODUCTION

A *water-surface profile* is a feature of the hydraulics of open channels which describes the variation of the water-surface elevation in the longitudinal direction (one dimension *x* in space), under a steady equilibrium flow condition. There are twelve (12) types of water-surface profiles, depending on the Froude number **F** and the ratio S_o / S_c , in which S_o = bottom slope, and S_c = critical slope. The critical slope S_c is equal to 1/8 of the Darcy-Weisbach friction factor **f**. Table 1 lists the twelve types of profiles (**Ponce, 2014**).

Table 1. The twelve (12) types of water-surface profiles.

Family		Character	Rule	$S_o > S_c$	$S_o = S_c$	$S_o < S_c$	$S_o = 0$	$S_o < 0$
I		Retarded (Backwater)	$1 > F^2 < (S_o / S_c)$	S_1	C_1	M_1	-	-
II	A	Accelerated (Drawdown)	$1 < F^2 < (S_o / S_c)$	S_2	-	-	-	-
	B	Accelerated (Drawdown)	$1 > F^2 > (S_o / S_c)$	-	-	M_2	H_2	A_2
III		Retarded (Backwater)	$1 < F^2 > (S_o / S_c)$	S_3	C_3	M_3	H_3	A_3

In this article, we describe the M_1 water-surface profile, a retarded (backwater) subcritical/subnormal profile, which may well be the most common profile in practice. The M_1 profile depicts flow in a mild channel, upstream of a reservoir (Fig. 1). We present two examples and show the respective online calculations.

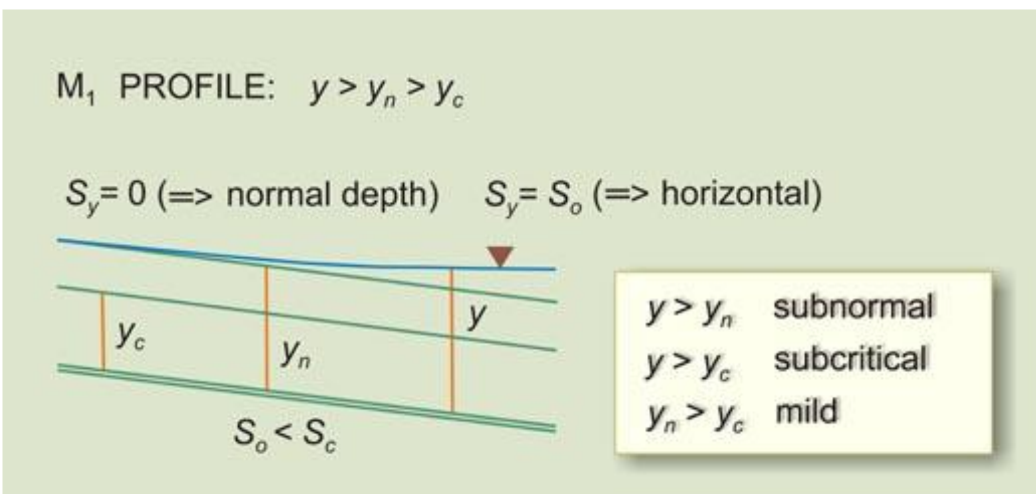


Fig. 1 M_1 water-surface profile.

2. GOVERNING EQUATION

Chow (1959) has presented the classical way of expressing the governing equation of steady, gradually varied flow. A more cogent, dimensionless presentation, focusing on critical slope, has been advanced by **Ponce (2014)** and is presented here.

In terms of critical slope, the general equation for flow-depth gradient d*y*/d*x* is:

$$\frac{dy}{dx} = \frac{S_o - (P / T)\,(T_c / P_c)\,S_c\,\boldsymbol{F}^2}{1 - \boldsymbol{F}^2} \qquad (1)$$

in which S_o = channel (bottom) slope, S_c = critical slope, P = wetted perimeter, T = channel top width, T_c = channel top width at critical flow, P_c = wetted perimeter at critical flow, and $\boldsymbol{F}$ = Froude number. The Froude number is defined as follows: $\boldsymbol{F} = v/(gD)^{1/2}$, in which v = mean flow velocity, g = gravitational acceleration, and D = hydraulic depth, in which $D = A / T$.

For $(P / T) = (P_c / T_c)$, Eq. 1 reduces to:

$$\frac{dy}{dx} = \frac{S_o - S_c\,\boldsymbol{F}^2}{1 - \boldsymbol{F}^2} \qquad (2)$$

For conciseness, the flow-depth gradient may be written as:

$$S_y = \frac{dy}{dx} \qquad (3)$$

Substituting Eq. 3 into Eq. 2, the flow-depth gradient is:

$$\frac{S_y}{S_c} = \frac{(S_o / S_c) - \boldsymbol{F}^2}{1 - \boldsymbol{F}^2} \qquad (4)$$

Equation 2, or Equation 4, its reduced form, is the steady gradually varied flow equation (Fig. 2). The depth gradient S_y is shown to be a function *only* of the following variables: (1) channel (bottom) slope S_o, (2) critical slope S_c , and (3) Froude number $\boldsymbol{F}$.

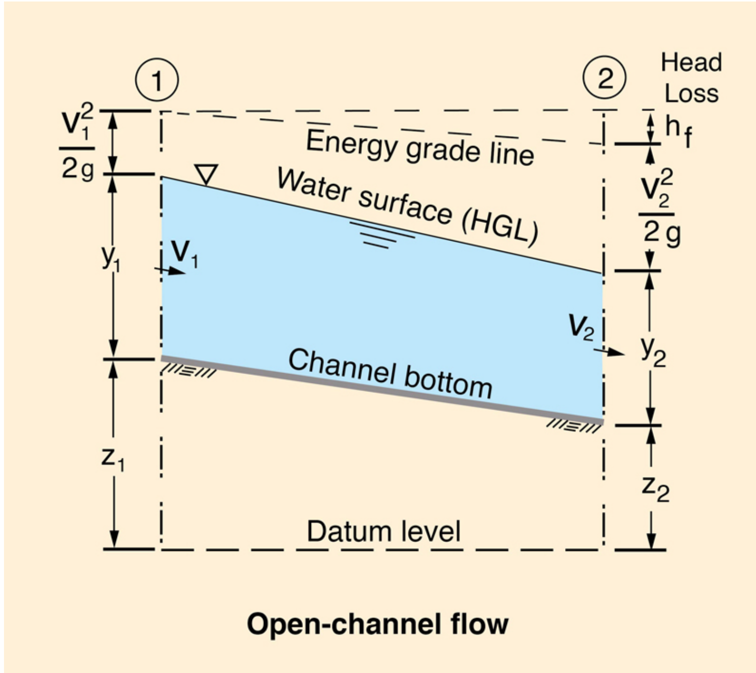


Fig. 2 Definition sketch for energy balance in open-channel flow.

3. ONLINE CALCULATION: EXAMPLE A

We pose an example of the calculation of an M_1 profile in a natural channel using the online calculator **ONLINE_WSPROFILES_21**. The following box shows the input data.

Example A: Input Data

- Discharge Q = 100 m^3/s
- Bottom width = 20 m
- Side slope z = 2 (z H : 1 V)
- Bottom slope S_o = 0.0005
- Manning's n = 0.03
- Normal depth at the downstream boundary y_d = 5 m
- Number of computation intervals: n = 100
- Number of tabular output intervals: m = 20

[Click on top of figure to expand]

[Click on top of figure to expand]

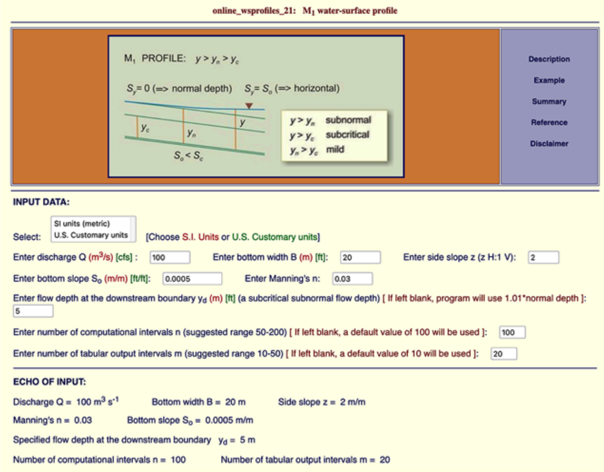


Fig. 3 (a) Example A: Input.

OUTPUT:

Computational depth interval Δy = 0.0204 m   Tabular output depth interval $(\Delta y)_t$ = 0.102 m

Normal depth y_n = 2.962 m   Normal-depth Froude number F_n = 0.268

k	Depth (m)	Area (m^2)	Velocity (m s^{-1})	Velocity head (m)	Specific head (m)	Wetted perimeter (m)	Hydraulic radius (m)	Friction slope (m/m)	Average slope (m/m)	Specific head difference (m)	Length increment (m)	Total length (m)
0	5	150	0.67	0.023	5.023	42.36	3.54	0.00007411	-	-	-	0
5	4.898	145.94	0.69	0.024	4.922	41.9	3.48	0.00008004	0.00007942	0.02	47.8	237.9
10	4.796	141.93	0.7	0.025	4.821	41.45	3.42	0.00008656	0.00008589	0.02	48.5	479.1
15	4.694	137.96	0.72	0.027	4.721	40.99	3.37	0.00009376	0.00009301	0.02	49.3	724.1
20	4.592	134.03	0.75	0.028	4.621	40.54	3.31	0.00010172	0.00010089	0.02	50.2	973.5
25	4.49	130.14	0.77	0.03	4.521	40.08	3.25	0.00011053	0.00010961	0.02	51.3	1227.8
30	4.389	126.29	0.79	0.032	4.421	39.63	3.19	0.00012031	0.00011929	0.02	52.5	1487.9
35	4.287	122.49	0.82	0.034	4.321	39.17	3.13	0.00013119	0.00013006	0.02	54	1754.7
40	4.185	118.72	0.84	0.036	4.221	38.71	3.07	0.00014333	0.00014206	0.02	55.7	2029.6
45	4.083	115	0.87	0.039	4.121	38.26	3.01	0.00015689	0.00015547	0.02	57.7	2313.9
50	3.981	111.32	0.9	0.041	4.022	37.8	2.94	0.00017209	0.00017050	0.02	60.2	2609.8
55	3.879	107.68	0.93	0.044	3.923	37.35	2.88	0.00018918	0.00018739	0.02	63.3	2920
60	3.777	104.08	0.96	0.047	3.824	36.89	2.82	0.00020843	0.00020641	0.02	67.2	3247.9
65	3.675	100.52	0.99	0.05	3.726	36.44	2.76	0.00023020	0.00022791	0.02	72.3	3598.8
70	3.573	97.01	1.03	0.054	3.628	35.98	2.7	0.00025489	0.00025229	0.02	79.2	3980
75	3.471	93.53	1.07	0.058	3.53	35.52	2.63	0.00028298	0.00028002	0.02	88.8	4403.3
80	3.37	90.1	1.11	0.063	3.432	35.07	2.57	0.00031507	0.00031169	0.019	103.2	4887.9
85	3.268	86.71	1.15	0.068	3.335	34.61	2.51	0.00035186	0.00034797	0.019	127.2	5470.6
90	3.166	83.36	1.2	0.073	3.239	34.16	2.44	0.00039420	0.00038972	0.019	174.3	6233.1
95	3.064	80.05	1.25	0.08	3.143	33.7	2.38	0.00044315	0.00043796	0.019	307.7	7436.8
100	2.962	76.79	1.3	0.086	3.048	33.25	2.31	0.00050000	0.00049396	0.019	3136.7	13114.5

Fig. 3 (b) Example A: Output.

Output. The results show that the (flow) depth at the downstream boundary (y_d = 5 m) will decrease gradually to the normal depth in the upstream end y_n = 2.962 m. The total distance, from downstream boundary to upstream end, is: L = 13,114.5 m.

4. ONLINE CALCULATION: EXAMPLE B

We pose an example of the calculation of an M_1 profile in a lined channel using the online calculator **ONLINE_WSPROFILES_21**. The following box shows the input data.

Example B: Input Data

- Discharge Q = 50 m^3/s
- Bottom width = 6 m
- Side slope z = 1 (z H : 1 V)
- Bottom slope S_o = 0.002
- Manning's n = 0.015
- Normal depth at the downstream boundary y_d = 5 m
- Number of computation intervals: n = 100
- Number of tabular output intervals: m = 10

[Click on top of figure to expand]

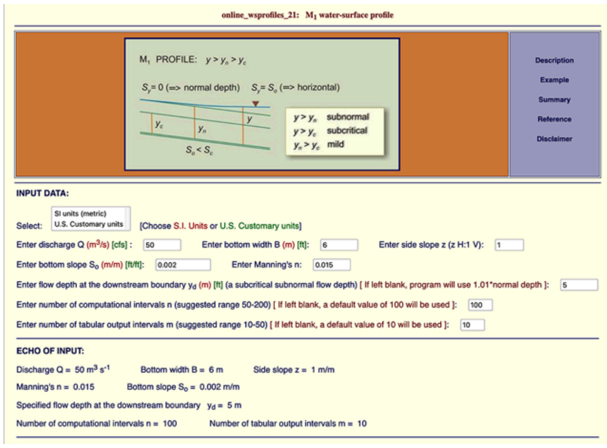
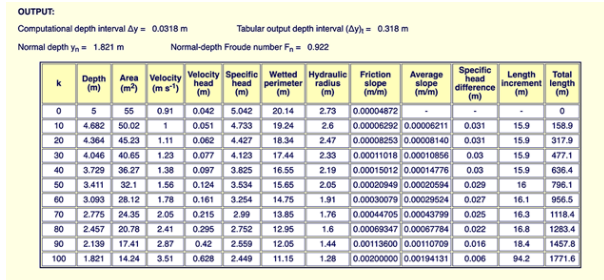


Fig. 4 (a) Example B: Input.

[Click on top of figure to expand]



OUTPUT:

Computational depth interval Δy = 0.0318 m Tabular output depth interval $(\Delta y)_t$ = 0.318 m

Normal depth y_n = 1.821 m Normal-depth Froude number F_n = 0.922

k	Depth (m)	Area (m²)	Velocity (m s⁻¹)	Velocity head (m)	Specific head (m)	Wetted perimeter (m)	Hydraulic radius (m)	Friction slope (m/m)	Average slope (m/m)	Specific head difference (m)	Length increment (m)	Total length (m)
0	5	55	0.91	0.042	5.042	20.14	2.73	0.00004872	-	-	-	0
10	4.682	50.02	1	0.051	4.733	19.24	2.6	0.00006292	0.00006211	0.031	15.9	158.9
20	4.364	45.23	1.11	0.062	4.427	18.34	2.47	0.00008253	0.00008140	0.031	15.9	317.9
30	4.046	40.65	1.23	0.077	4.123	17.44	2.33	0.00011018	0.00010856	0.03	15.9	477.1
40	3.729	36.27	1.38	0.097	3.825	16.55	2.19	0.00015012	0.00014776	0.03	15.9	636.4
50	3.411	32.1	1.56	0.124	3.534	15.65	2.05	0.00020949	0.00020594	0.029	16	796.1
60	3.093	28.12	1.78	0.161	3.254	14.75	1.91	0.00030079	0.00029524	0.027	16.1	956.5
70	2.775	24.35	2.05	0.215	2.99	13.85	1.76	0.00044705	0.00043799	0.025	16.3	1118.4
80	2.457	20.78	2.41	0.295	2.752	12.95	1.6	0.00069347	0.00067784	0.022	16.8	1283.4
90	2.139	17.41	2.87	0.42	2.559	12.05	1.44	0.00113600	0.00110709	0.016	18.4	1457.8
100	1.821	14.24	3.51	0.628	2.449	11.15	1.28	0.00200000	0.00194131	0.006	94.2	1771.6

Fig. 4 (b) Example B: Output.

Output. The results show that the (flow) depth at the downstream boundary (y_d = 5 m) will decrease gradually to the normal depth in the upstream end y_n = 1.821 m. The total distance, from downstream boundary to upstream end, is: L = 1,771.6 m.

5. SUMMARY

An online calculation of an M_1 open-channel flow water-surface profile is shown in detail. Two examples using the script **ONLINE_WSPROFILES_21** demonstrate the utility of using this online digital tool for the accurate and expedient calculation of an M_1 water-surface profile.

REFERENCES

Chow, V. T. 1959. *Open-channel hydraulics*. McGraw-Hill, Inc, New York, NY.

Ponce, V. M. 2014b. **Fundamentals of Open-channel Hydraulics**. Online textbook.
https://ponce.sdsu.edu/openchannel/index.html

231025